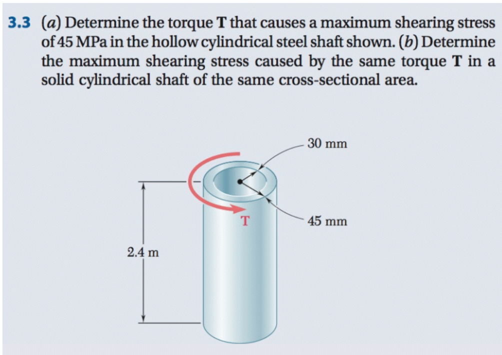
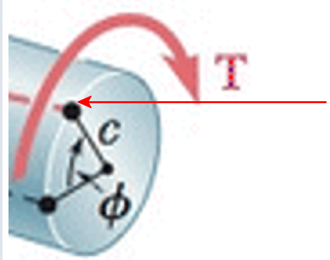
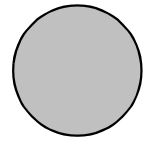
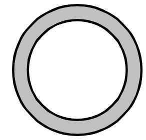




$$\tau_{max} = \frac{Tc}{J}$$



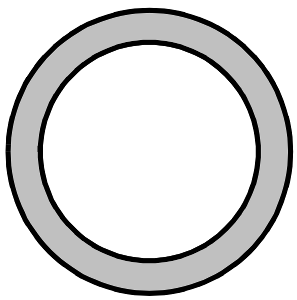
$$J = \frac{1}{2}\pi c^4$$



$$J = \frac{1}{2}\pi\left(c_o^4 - c_i^4\right)$$

This problem requires that you examine the effect of changing the Polar Moment of Inertia (J)

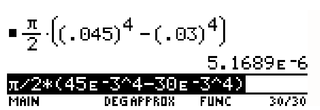
Original



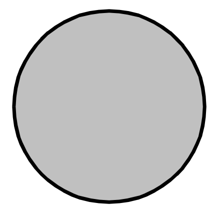
$$J = \frac{1}{2}\pi\left(c_o^4 - c_i^4\right)$$

$$= \frac{1}{2}\pi\left[\left(45\times10^{-3}m\right)^4 - \left(30\times10^{-3}m\right)^4\right]$$

$$5.17\times10^{-6}m^4$$



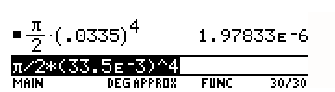
New[1]



$$J = \frac{1}{2}\pi c^4$$

$$= \frac{1}{2}\pi\left(33.5\times10^{-3}m\right)^4$$

$$= 1.98\times10^{-6}m^4$$



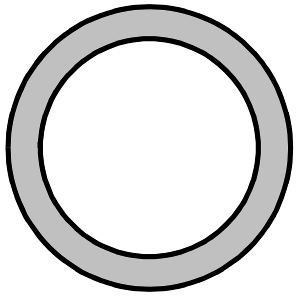
The maximum torque can be determined as follows.

$$\tau = \frac{Tc}{J}$$

$$T = \frac{\tau J}{c}$$

[1]Please note that this is based on maintaining cross sectional area. Please contact me if the calculation of the new radius 33.5mm is unclear.

Original



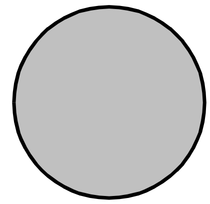
$$T = \frac{\tau J}{c}$$

$$= \frac{\tau J}{c} = \frac{\left(45 \times 10^{6} \frac{N}{m^2}\right)\left(5.17 \times 10^{-6} m^4\right)}{\left(45 \times 10^{-3} m\right)}$$

$$= 5.2 \times 10^{3}\ N \cdot m$$

```
■ 45000000.·5.17E-6
  -----------------
        .045
                          5.17E3
(45E6*5.17E-6)/45E-3
MAIN        DEG APPROX    FUNC    30/30
```

New



$$T = \frac{\tau J}{c}$$

$$= \frac{\tau J}{c} = \frac{\left(45 \times 10^{6} \frac{N}{m^2}\right)\left(1.98 \times 10^{-6} m^4\right)}{\left(33.5 \times 10^{-3} m\right)}$$

$$= 2.7 \times 10^{3}\ N \cdot m$$

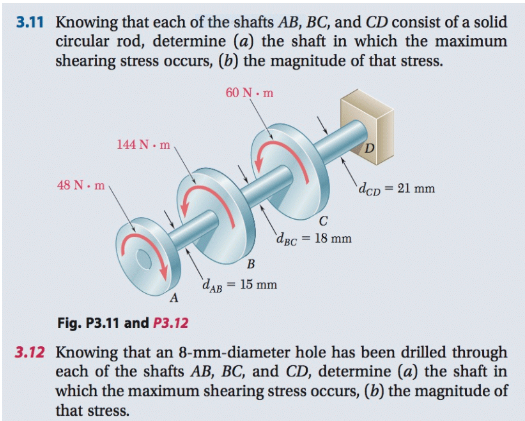
```
■ 45000000.·1.98E-6
  -----------------
       .0335
                        2.6597E3
(45E6*1.98E-6)/33.5E-3
MAIN        DEG APPROX    FUNC    30/30
```

The shear stress at the surface of the new cylinder is:

$$\tau = \frac{Tc}{J} = \frac{\left(5.2 \times 10^{3}\ N \cdot m\right)\left(33.5 \times 10^{-3} m\right)}{\left(1.98 \times 10^{-6} m^4\right)}$$

$$= 88 \times 10^{6} \frac{N}{m^2}$$

```
■ 5200.·.0335
  -----------       87.9798E6
   1.98E-6
(5.2E3*33.5E-3)/1.98E-6
MAIN        DEG APPROX    FUNC    30/30
```



3.11 Knowing that each of the shafts *AB*, *BC*, and *CD* consist of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

Fig. P3.11 and ***P3.12***

3.12 Knowing that an 8-mm-diameter hole has been drilled through each of the shafts *AB*, *BC*, and *CD*, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

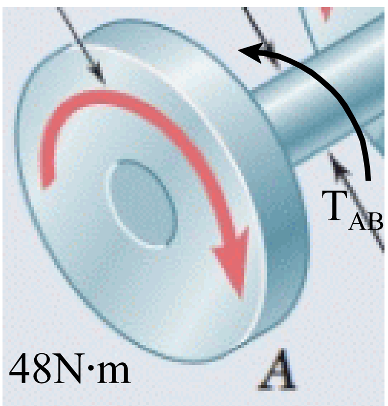
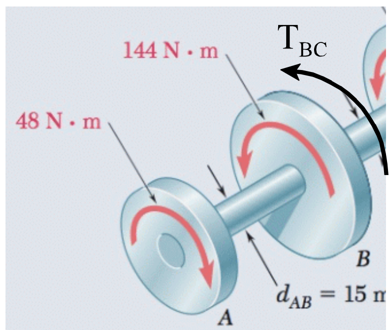
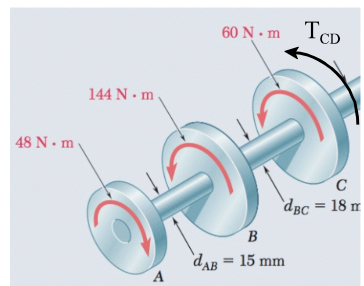
Given that the maximum shear stress in each rod, that is the stress at the surface is:

$$\tau = \frac{Tc}{J} = T\frac{2}{\pi c^3} = T\frac{16}{\pi d^3}$$

Thus, $\frac{16}{\pi d^3}$ is a geometrical factor for each of the shafts.

For AB this is $\frac{16}{\pi\left(15\times 10^{-3}m\right)^3} = 1.5\times 10^6 m^{-3}$.

For BC this is $870\text{x}10^3 m^{-3}$, and for CD this is $550\text{x}10^3 m^{-3}$.

 The determination of T_{AB} is based on the Free Body Diagram for the Section. Because the sum of the moments about an axis must be 0, T_{AB} must equal 48 N·m.	 The determination of T_{BC} is based on the Free Body Diagram for the Section. Because the sum of the moments about an axis must be 0, T_{BC} must equal 96 N·m.	 The determination of T_{CD} is based on the Free Body Diagram for the Section. Because the sum of the moments about an axis must be 0, T_{CD} must equal 156 N·m

The shear stresses for the three shafts are:
For AB = $72\text{x}10^6$ N/m^2
For BC = $84\text{x}10^6$ N/m^2
For CD = $86\text{x}10^6$ N/m^2

3.48 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both *AB* and *CD*. It is further required that $\tau_{max} \le 60$ MPa and that the angle ϕ_D through which end *D* of shaft *CD* rotates not exceed 1.5°. Knowing that $G = 77.2$ GPa, determine the required diameter of the shafts.

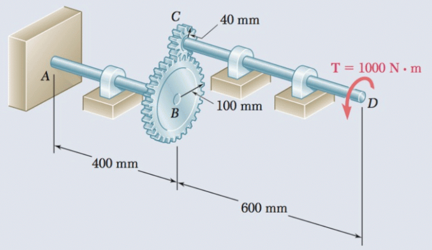
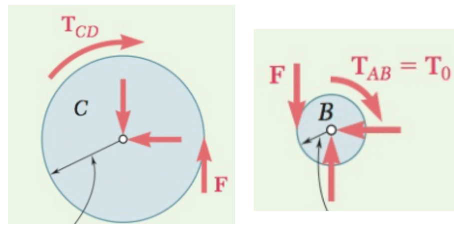


Fig. P3.48

It is necessary to calculate the torque in each shaft. T_{CD} can be determined as the sum of the moments must equal 0. T_{CD} =1000N·m. However the total force at the interface between the gears must be 0. This was shown in our exploration of Sample Problem 3.4.



This means that,

From Class Notes Torsion 2

S

$$T_{AB} = r_B F$$

$$T_{CD} = r_c F$$

$$\frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_c}$$

$$T_{CD} = T_{AB}\frac{r_c}{r_B} = 2.5T_{AB}$$

ince the shafts are going to have the same diameter it is only necessary to determine the required diameter for CD. Based on the formula for shear stress at the surface of a shaft.

$$\tau_{max} = \frac{Tc}{J}$$

The following formula can be determined

$$\frac{J}{c} = \frac{T}{\tau_{max}} = \frac{1}{2}\pi c^3$$

This means,

$$c^3 = \frac{2}{\pi}\frac{T}{\tau_{max}}$$

c can then be calculated as shown,

$$c = \sqrt[3]{\frac{2}{\pi}\frac{T}{\tau_{\max}}} = \sqrt[3]{\frac{2}{\pi}\frac{1000 N \cdot m}{60 \times 10^{6}\ \frac{N}{m^2}}} = 22 \times 10^{-3}\, m$$

The angle of twist at D will be

$$\phi_D = \phi_C + \phi_{C/D}$$

$$\phi_C = \frac{r_C}{r_B}\phi_B$$

$$= \frac{r_C}{r_B}\frac{T_{AB} L_{AB}}{JG}$$

$$= \frac{r_C}{r_B}\frac{r_B}{r_C}\frac{T_0 L_{AB}}{JG}$$

$$\phi_D = \frac{T_0 L_{AB}}{JG} + \frac{T_0 L_{CD}}{JG}$$

$$= \frac{T_0}{JG}\left(L_{AB} + L_{CD}\right)$$

$$J = \frac{T_0}{\phi_D G}\left(L_{AB} + L_{CD}\right)$$

$$c^4 = \frac{2}{\pi}\frac{T_0}{\phi_D G}\left(L_{AB} + L_{CD}\right)$$

$$c = \sqrt[4]{\frac{2}{\pi}\frac{T_0}{\phi_D G}\left(L_{AB} + L_{CD}\right)}$$

$$= \sqrt[4]{\frac{2}{\pi}\frac{1000 N \cdot m}{\left(26 \times 10^{-3} rad\right)\left(77.2 \times 10^{9}\ \frac{N}{m^2}\right)}\left(400 \times 10^{-3} m + 600 \times 10^{-3} m\right)}$$

$c=23.8x10^{-3}m$

The required diameter is 48mm ($48x10^{-3}m$)

3.38 The aluminum rod *AB* (G = 27 GPa) is bonded to the brass rod *BD* (G = 39 GPa). Knowing that portion *CD* of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at *A*.

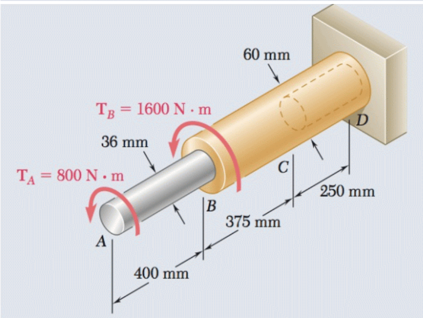


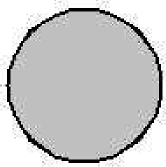
Fig. P3.38

The angle of twist at A will be,

$$\phi_A = \phi_{D/C} + \phi_{C/B} + \phi_{B/A}$$
$$= \frac{T_{CD}L_{CD}}{J_{CD}G_{CD}} + \frac{T_{BC}L_{BC}}{J_{BC}G_{BC}} + \frac{T_{AB}L_{AB}}{J_{AB}G_{AB}}$$

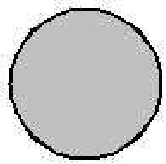
This means we need to determine the torques and polar moments of inertia J for each section.

J for AB



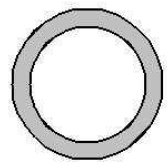
$$J = \frac{1}{2}\pi c^4$$
$$= \frac{1}{2}\pi\left(18\times10^{-3}\,m\right)^4$$
$$= 165\times10^{-9}\,m^4$$

J for BC



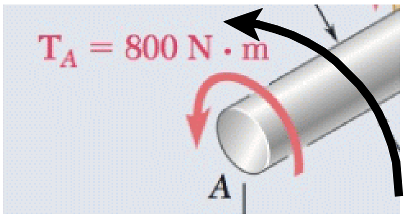
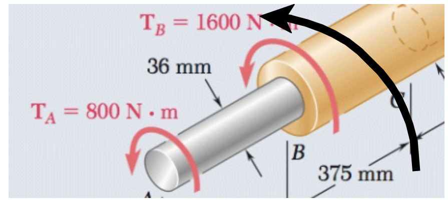
$$J = \frac{1}{2}\pi c^4$$
$$= \frac{1}{2}\pi\left(30\times10^{-3}\,m\right)^4$$
$$= 1.27\times10^{-6}\,m^4$$

J for CD



$$J = \frac{1}{2}\pi\left(c^4 - c_i^4\right)$$
$$= \frac{1}{2}\pi\left[\left(30\times10^{-3}\,m\right)^4 - \left(20\times10^{-3}\,m\right)^4\right]$$
$$= 1.02\times10^{-6}\,m^4$$

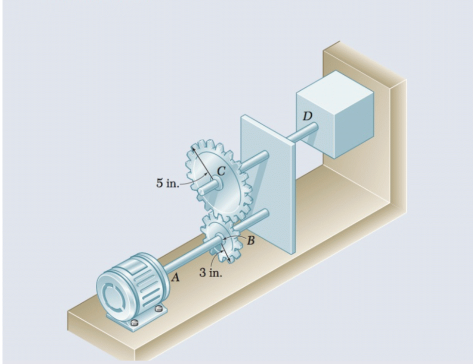
The torques within each member must be determined based on the total moment about any axis must be zero. These will be determined by sectioning as shown below.

For AB	For BC and CD
 T_{AB}	 T_{BD}

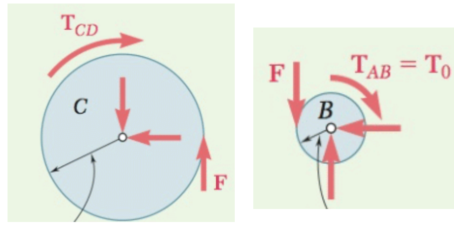
Thus the three torques are

- T_{AB}=800N·m
- T_{BC}=2400N·m
- T_{CD}=2400N·m

$$\begin{aligned}
\phi_A &= \phi_{D/C} + \phi_{C/B} + \phi_{B/A} \\
&= \frac{T_{CD}L_{CD}}{J_{CD}G_{CD}} + \frac{T_{BC}L_{BC}}{J_{BC}G_{BC}} + \frac{T_{AB}L_{AB}}{J_{AB}G_{AB}} \\
&= \frac{(2400N\cdot m)(250\times10^{-3}m)}{(1.02\times10^{-6}m^4)(39\times10^9\,\frac{N}{m^2})} + \frac{(2400N\cdot m)(375\times10^{-3}m)}{(1.27\times10^{-6}m^4)(39\times10^9\,\frac{N}{m^2})} + \frac{(800N\cdot m)(400\times10^{-3}m)}{(0.165\times10^{-6}m^4)(27\times10^9\,\frac{N}{m^2})} \\
&= 0.105 rad = 6\deg
\end{aligned}$$


3.76 The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A* operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that each shaft has a diameter of 1 in., determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *CD*.

The total force at the interface between the gears must be 0. This was shown in our exploration of Sample Problem 3.4.



From Class Notes Torsion 2

This means that,

$$T_{AB} = r_B F$$
$$T_{CD} = r_c F$$
$$\frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_c}$$
$$T_{CD} = T_{AB}\frac{r_c}{r_B} = T_{AB}\frac{5in}{3in} = 1.7T_{AB}$$

Since the torque in AB is due to the power.

$$T = \frac{P}{2\pi f}$$
$$= \frac{1600hp}{2\pi(1260rpm)} = \frac{52.8\times 10^6 \frac{ft\cdot lb}{\min}}{2\pi\left(1260\frac{1}{\min}\right)}$$
$$6.57\times 10^3\, ft\cdot lb$$

Because the diameter of both rods are 1 inch, J and c will be equal for both. Since,

$$\tau = \frac{Tc}{J}$$
$$\tau_{CD} = 1.7\tau_{AB}$$

For shaft AB,

$$\begin{aligned} c^3 &= \frac{2}{\pi}\frac{T}{\tau_{\max}} \\ \tau_{\max} &= \frac{2}{\pi}\frac{T}{c^3} \\ &= \frac{2}{\pi}\frac{74.8\times10^3\, in\cdot lb}{(0.5in)^3} \\ &= 380\times10^3\, psi \end{aligned}$$

For shaft CD this will be $646\text{x}10^3$psi